

Gravitational Lorentz Violations and Adjustment of the Cosmological Constant in Asymmetrically Warped Spacetimes

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Abstract

We investigate spacetimes in which the speed of light along flat 4D sections varies over the extra dimensions due to different warp factors for the space and the time coordinates (“asymmetrically warped” spacetimes). The main property of such spaces is that while the induced metric is flat, implying Lorentz invariant particle physics on a brane, bulk gravitational effects will cause apparent violations of Lorentz invariance and of causality from the brane observer’s point of view. An important experimentally verifiable consequence of this is that gravitational waves may travel with a speed different from the speed of light on the brane, and possibly even faster. We find the most general spacetimes of this sort, which are given by AdS–Schwarzschild or AdS–Reissner–Nordström black holes, assuming the simplest possible sources in the bulk. Due to the gravitational Lorentz violations these models do not have an ordinary Lorentz invariant effective description, and thus provide a possible way around Weinberg’s no-go theorem for the adjustment of the cosmological constant. Indeed we show that the cosmological constant may relax in such theories by the adjustment of the mass and the charge of the black hole. The black hole singularity in these solutions can be protected by a horizon, but the existence of a horizon requires some exotic energy densities on the brane. We investigate the cosmological expansion of these models and speculate that it may provide an explanation for the accelerating Universe, provided that the timescale for the adjustment is shorter than the Hubble time. In this case the accelerating Universe would be a manifestation of gravitational Lorentz violations in extra dimensions.

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1 Introduction

The physics of extra dimensions has recently attracted renewed interest, mainly for the following three reasons:

- The existence of extra dimensions could provide a stable large hierarchy between the scale of particle physics (the TeV scale) and the scale of gravity (the Planck scale) [1, 2].
- One can obtain the 4D Einstein equations from a higher dimensional setup without compactification or a need for a stabilization mechanism [2–4], and the expansion of the brane driven by matter follows the usual 4D Friedmann equations [5].
- One may hope that the cosmological constant problem could be partly resolved due to extra dimensional physics [6–13].

Randall and Sundrum (RS) studied spacetimes with a single “warped” extra dimension, for which the metric is of the form

$$ds^2 = e^{-A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (1.1)$$

They showed that such geometries can have interesting consequences for particle physics and gravity. In these scenarios the standard model fields are usually assumed to live at a particular point in the extra dimension, called a 3-brane. Metrics of the form (1.1) can reproduce 4D Einstein gravity on the brane without compactification, and can produce a large hierarchy between the scales of particle physics and gravity due to the appearance of the warp factor.

In this paper we study the most general backgrounds with one extra dimension, assuming 3D rotational invariance is still maintained⁽¹⁾ (and with the additional assumption that the only sources in the bulk are a bulk cosmological constant and a $U(1)$ gauge field). The metric of such backgrounds can be generically written as,

$$ds^2 = -a^2(r, t) dt^2 + b^2(r, t) d\vec{x}^2 + c^2(r, t) dr^2. \quad (1.2)$$

However, when the sources are restricted to a bulk cosmological constant and a bulk gauge field, a five dimensional version of Birkhoff’s theorem holds, and such a metric can always be transformed into the form,

$$ds^2 = -h(r)dt^2 + r^2 d\Sigma^2 + h(r)^{-1} dr^2. \quad (1.3)$$

Here $d\Sigma$ is the unit metric of the 3D sections $r, t=\text{const.}$, and $h(r)$ describes a black hole spacetime. Examining (1.3) one can see that the novel property of such a general 5D spacetime compared to conventional warped background (1.1) is that the warp factors for the space and time components of the 4D sections $r=\text{const.}$ are generically different. We will refer to metrics of this form as “asymmetrically warped”. The induced metric at the 4D sections may still be flat, implying that (up to quantum gravitational corrections) particle physics on the brane will see a Lorentz invariant spacetime. However, since every 4D section of the metric (1.3) will have a differently defined Lorentz symmetry (one needs to rescale the time coordinate

⁽¹⁾The breaking of 3D rotational invariance in the bulk would transmit its breaking in gravitational interactions on the brane in contradiction with the isotropy of the cosmic microwave background.

differently at different points along the extra dimension to maintain a speed of light $c = 1$ along each section), the spacetime (1.3) globally violates 4D Lorentz invariance, leading to apparent violations of Lorentz invariance from the brane observer's point of view due to bulk gravity effects. These Lorentz violations however produce different effects from explicit Lorentz violations that are sometimes introduced (see for example [14]) in particle physics. In fact, since at the classical level they only affect gravity, the most striking consequence of this setup would be that the speed of gravitational waves would be different from the speed of light, which could cause apparent violations of causality from the 4D brane observer's point of view. This possibility has already been pointed out by Kälbermann and Halevi [15], Chung and Freese [16], Ishihara [17] and Chung, Kolb and Riotto [18]. Note also that the existence of different speeds of propagation for gravitational and electromagnetic interactions has some common features with four dimensional theories of gravity with two light cones as proposed in [19]. In asymmetrically warped spacetimes the reason for the different speeds of propagation for graviton and photon is that in the background (1.3) the speed of light along the brane is changing as one is moving along the extra dimension. Thus this setup is analogous to a medium with a changing index of refraction. If the speed of light away from the brane is increasing, then by Fermat's principle the geodesic between two points on the brane will bend into the bulk, and the gravitational wave which is not forced to propagate on the brane will arrive faster than the light signal which is stuck to the brane. In fact, this difference in the speed of electromagnetic and gravitational waves could be viewed as a generic prediction of extra dimensions, which can be experimentally verified [18,19] once gravitational waves are observed by LIGO, VIRGO or LISA.

It has been widely recognized starting with the work of Rubakov and Shaposhnikov [6], that extra dimensions provide a new approach to the cosmological constant problem: with the presence of extra dimensions one no longer needs to ensure the vanishing of the 4D vacuum energy on the brane in order to obtain a static flat brane. The non-vanishing brane tension can be balanced by the bulk curvature, exactly as it happens for example in the Randall-Sundrum model. However, in the RS background a tuning between the bulk cosmological constant and the brane tension is required in order to achieve this balance. The second important consequence of the metric (1.3) is that it contains parameters in addition to the ones contained in the solution discussed by RS, namely the mass, charge and location of the black hole with respect to the brane, which can be thought of as integration constants for the most general solution in the bulk. Thus one may hope that the finetuning required in the original RS setup in order to ensure the vanishing of the 4D effective cosmological constant could be eliminated. This would be similar in spirit to the “self-tuning” models [7,8], where a bulk scalar is coupled to the tension of the visible brane, and allows the transfer of curvature into the bulk if the brane tension is adjusted, so as to maintain flatness of the brane. In our case, the adjustment mechanism would imply that once the brane tension goes through a phase transition, the black hole would adjust its mass, charge and location in order to balance the new stress tensor on the brane. One important ingredient in the model of [7,8] was that for a particular coupling of the bulk scalar field to the brane tension the only maximally symmetric solution to the equations of motion was the flat brane solution, thus making it plausible that the time dependent solution may relax to the flat brane. In the black hole backgrounds this feature is automatically present, without having to tune a coupling: for generic equations of state we will show that the only maximally symmetric brane sections correspond to the flat branes. For simplicity we will

assume that the spacetime is \mathbb{Z}_2 symmetric around the brane.

There are two reasons why this approach based on bulk black hole spacetimes may be preferable compared to the scenario in [7, 8]:

- Due to the gravitational Lorentz violations explained above, there is no ordinary Lorentz invariant low-energy effective 4D description of this model, and thus Weinberg’s no-go theorem [20] for 4D adjustment mechanisms of the cosmological constant can not be applied here. This reasoning is similar to that used to argue that models of quasilocalized gravity [21] may be relevant to the cosmological constant problem.
- One may hope that due to the appearance of black holes, the unavoidable [11] naked singularities appearing in [7, 8] would be replaced by black hole singularities shielded by a horizon.

We will find that it is in fact possible to protect the singularities by a horizon, and the spacetime can be cut at the location of the horizon without reintroducing finetuning into the theory (that is, without the need to regulate the spacetime with, for example, an additional brane as in [10]). However, we find that such horizons are consistent with the presence of a \mathbb{Z}_2 symmetric brane only in the case of charged black holes, and require the presence of some exotic (but not fine-tuned) energy density on the brane. Of course this exotic energy density is not generically required for an asymmetrically warped background, only for an adjusting solution with a horizon.

In order to gain some insight on how the effective gravity theory would behave from the brane observer’s point of view, we investigate some of the basic features of the cosmology of these models. We find that the Friedmann equation does contain the ordinary expansion term, but there are additional contributions due to the mass and charge of the black hole, which was also pointed out in [22, 23]. These terms could be useful in explaining the apparent acceleration of the Universe [24], however in order to obtain a viable cosmology and solve the cosmological constant problem at the same time the adjustment timescale of the mass and charge of the black hole must be shorter than the Hubble time.

The paper is organized as follows: in Section 2 we first show what the generic form of the solutions in the bulk is, and then introduce the brane by solving the junction conditions. We examine the fine-tuning of the parameters and the existence of a horizon. We close Section 2 by showing that there are no other maximally symmetric solutions. Section 3 is devoted to the physical consequences of the asymmetrically warped spacetimes. We calculate the geodesics and the speed of gravitational waves, then show how a graviton zero mode would look like in these theories, and finally consider the cosmology of these models.

2 Flat branes in black hole backgrounds

Our aim is to find general 4D Lorentz invariant brane (“flat brane”) solutions to Einstein’s equations in five dimensions, and to investigate the following issues: is it possible to find flat brane solutions without naked singularities, without fine tuning the parameters of the theory, and what would the general properties of such spaces be? In this section we will first discuss the form of the solution in the bulk, and then impose the condition for a static brane. We will see that the pure gravity theory will always require a fine-tuning between the parameters of

the theory for a solution to exist. However, if one also includes a gauge field in the bulk, the situation changes, and one can find flat solutions for a range of parameters with no tuning. However, for these solutions one of the following two limitations will apply: either there is no horizon but a naked singularity, or if one does want to ensure the existence of a horizon one has to assume the presence of some exotic matter on the brane.

2.1 The solution in the bulk

We model the 5D bosonic sector of the theory by a graviton and a cosmological constant Λ_{bk} as in the Randall–Sundrum model, and in addition a $U(1)$ gauge field propagating in the bulk — we will assume that the 4D matter living on the brane is neutral under this gauge symmetry. So the action describing the dynamics of the system is given by,⁽¹⁾

$$\mathcal{S} = \int d^5x \sqrt{|g_5|} \left(\frac{1}{2\kappa_5^2} R - \frac{1}{4} F_{MN} F^{MN} - \Lambda_{bk} \right) + \int d^4x \sqrt{|g_4|} \mathcal{L}_{\text{mat.}}(g_{\mu\nu}, \psi^{(m)}) \quad (2.1)$$

where g_5 and g_4 are, respectively, the determinants the 5D metric g_{MN} and the 4D metric induced on the brane, $g_{\mu\nu}$; $\psi^{(m)}$ denote some matter fields localized on the brane and $\mathcal{L}_{\text{mat.}}$ describes their interactions, and we will assume that this matter can be described as a perfect fluid of energy density ρ and pressure p ; κ_5^2 defines the 5D Planck scale. Note that the brane Lagrangian \mathcal{L}_{mat} includes the brane tension $\rho = -p$.

We will assume that the solution is homogeneous and isotropic in the three spatial directions of the brane. The general ansatz is,

$$ds^2 = -n^2(\tilde{t}, \tilde{r}) d\tilde{t}^2 + a^2(\tilde{t}, \tilde{r}) d\Sigma_k^2 + b^2(\tilde{t}, \tilde{r}) d\tilde{r}^2, \quad (2.2)$$

where $d\Sigma_k^2 = d\sigma^2/(1 - kL^{-2}\sigma^2) + \sigma^2 d\Omega_2^2$ is the metric of the spatial 3-sections, with curvature parameter k , L being a parameter with dimension of length that will be set to the length scale given by the cosmological constant in the bulk. The brane is located at $\tilde{r} = 0$. The 3D sections $\tilde{r}, \tilde{t} = \text{const.}$ are either a plane ($k = 0$), a unit sphere ($k = 1$) or a unit hyperboloid ($k = -1$). The brane will be expanding or contracting as long as the scale factor $a(\tilde{t}, 0)$ explicitly depends on time. Some of the features of metrics of the above form (2.2) have also been independently investigated in Refs. [15, 16, 18],

In the case when there is no gauge field in the bulk, Kraus [25] and later Bowcock *et al.* [26] have shown that a generalization of Birkhoff's theorem [27] holds in the bulk, which implies that there exists a system of coordinates where the 5D metric is static while, in general, the brane is moving and expanding. In this system of coordinates, the most general metric is,

$$ds^2 = -h(r) dt^2 + l^{-2} r^2 d\Sigma_k^2 + h(r)^{-1} dr^2, \quad (2.3)$$

where

$$h(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2}; \quad l^{-2} = -\frac{1}{6}\kappa_5^2 \Lambda_{bk}. \quad (2.4)$$

⁽¹⁾ Our conventions correspond to a mostly positive Lorentzian signature $(- + \dots +)$ and the definition of the curvature in terms of the metric is such that a Euclidean sphere has positive curvature. Bulk indices will be denoted by Latin indices $(M, N \dots)$ and brane indices by Greek indices $(\mu, \nu \dots)$.

The new coordinates r and t are functions of the brane-based coordinates \tilde{r} and \tilde{t} and thus the location of the brane is parametrized by $r = R(t)$ and is solution of a differential equation we will give in the next section. The bulk geometry describes an AdS–Schwarzschild hole (a “black wall” for $k = 0$) located at $r = 0$ and spreading in the three other spatial directions. The parameter μ is interpreted as the mass (in units where the five dimensional Planck scale is equal to one) of the black hole. When $\mu = 0$, the metric simply describes 5D Anti-de Sitter space and for a non-vanishing positive μ the $r = 0$ singularity is hidden behind a horizon $r = r_h > 0$ where the metric becomes degenerate $h(r_h) = 0$.

Birkhoff’s theorem can also be generalized when the gauge field is turned on. This way we obtain the AdS–Reissner–Nordström metric (see also [22]), which has the same form as (2.3), except that now the function h also depends on the charge Q of the black hole:

$$h(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2} + \frac{Q^2}{r^4} \quad (2.5)$$

The non-vanishing component of the bulk field strength tensor F of the gauge field satisfies the Bianchi identities and is given by,

$$F_{tr} = \frac{\sqrt{6}Q}{\kappa_5 r^3}. \quad (2.6)$$

Later we will mainly be interested in the case $k = 0$ because in that case the (3+1)-dimensional sections $r=\text{constant}$ are flat. If the charge of a Reissner–Nordström black hole is too large, there is no horizon. In our solutions with $k = 0$, the existence of an inner and an outer horizon gives the following upper bound on the charge

$$Q^4 < \frac{4}{27}\mu^3 l^2. \quad (2.7)$$

This follows from the fact that the position of a horizon corresponds to the square root of a positive root of the cubic function $f_3(x) = x^3/l^2 - \mu x + Q^2$. As long as its discriminant is negative, f_3 will have three roots and, as we will argue in Section 2.4, two of them are positive and thus they are associated to two horizons.

2.2 Matching at the brane

We will assume that a codimension one brane separates two regions of the above discussed 5D black hole spacetimes. That is, just like in the original RS scenario, we are gluing two slices of the metric together at the position of the brane. In order for this to be possible, one has to satisfy the Israel junction conditions (also known as “the jump equations”) at the brane. We will restrict ourselves to solutions with a \mathbb{Z}_2 symmetry between the two sides of the brane which means that we study the expansion of a brane located at a fixed point of a \mathbb{Z}_2 orbifold. A simple way of defining a \mathbb{Z}_2 symmetric spacetime by gluing patches together has been explained in Ref. [28]. The idea is that for a metric of the form

$$ds^2 = -A^2(r)dt^2 + B^2(r)d\Sigma_\kappa^2 + C^2(r)dr^2 \quad (2.8)$$

one can find another solution using the fact that one still has reparametrization invariance in the coordinate r , and thus $\hat{A}(\hat{r}) = A(f(\hat{r}))$, $\hat{B}(\hat{r}) = B(f(\hat{r}))$, $\hat{C}(\hat{r}) = \pm C(f(\hat{r}))f'(\hat{r})$ still solves

the Einstein equations. Thus one can use this invariance to generate new solutions by gluing a solution to its image under the reparametrization. A particularly simple way of doing this is by picking $f(r) = r_0^2/r$, and the identification

$$\begin{aligned} \text{for } r \leq r_0 \quad & A(r) = A_0(r), \quad B(r) = B_0(r), \quad C(r) = C_0(r); \\ \text{for } r \geq r_0 \quad & A(r) = A_0(r_0^2/r), \quad B(r) = B_0(r_0^2/r), \quad C(r) = C_0(r_0^2/r) \frac{r_0^2}{r^2}, \end{aligned} \quad (2.9)$$

which will automatically ensure the continuity of the metric functions, and give an $r \leftrightarrow r_0^2/r$ \mathbb{Z}_2 symmetry of the metric. This is analogous to the $y \leftrightarrow -y$ \mathbb{Z}_2 symmetry in the Randall-Sundrum model, and in fact coincides with it upon doing the coordinate transformation $r = l e^{-y/l}$. From now on we will always assume the existence of such a \mathbb{Z}_2 symmetry in order to simplify the equations and we will apply the previous method on the metric defined by (2.5) between the black hole and the brane.

In a general case, the brane will not be static, but instead it will be moving in the bulk spacetime, expressing the fact that the observed brane Universe is expanding (or shrinking). The junction conditions for this case have been worked out in detail in Refs. [25, 26, 29, 30]. It is assumed that the brane follows a trajectory $R(\tau)$ in the above bulk spacetime. The proper time τ as observed on the brane is defined by the equation

$$\dot{t}^2 h(R(\tau)) - \dot{R}^2(\tau) h^{-1}(R(\tau)) = 1, \quad (2.10)$$

which ensures that the induced metric on the brane will be of the Friedmann–Robertson–Walker form $ds_{ind}^2 = -d\tau^2 + R(\tau)^2 d\Sigma_k^2$. Keeping the interior (the region next to $r = 0$) of the BH space-time to prevent the volume of the extra dimension to diverge,⁽²⁾ one then obtains the junction conditions [26]:⁽³⁾

$$\rho = \frac{6}{\kappa_5^2 R} \sqrt{h(R) + \dot{R}^2}, \quad (2.11)$$

$$\frac{2}{3}\rho + p = -\frac{2\ddot{R} + h'(R)}{\kappa_5^2 \sqrt{h(R) + \dot{R}^2}}, \quad (2.12)$$

where ρ is the energy density of the brane, while p is its pressure. That is, the energy momentum tensor on the brane is given by $T_B^A = \text{diag}(-\rho, p, p, p, 0)\delta(\sqrt{g_{rr}}(r - R(\tau)))$. As long as the brane is moving, *i.e.*, $\dot{R} \neq 0$, one combination of these jump equations is just equivalent to the energy conservation equation on the brane,

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0, \quad (2.13)$$

and the remaining equation simply reads,

$$\frac{\dot{R}^2}{R^2} = \frac{1}{36}\kappa_5^4 \rho^2 - \frac{h(R)}{R^2}. \quad (2.14)$$

⁽²⁾If one wants to keep the exterior (the region next to $r = +\infty$) of the BH space-time, both signs of the jump equations (2.11)–(2.12) have to be flipped. We will discuss this issue in more detail at the end of Section 2.4

⁽³⁾Similar differential equations have been obtained in [31] in a different context where the brane just probes the embedding space-time and moves along a geodesic. The following equations ensure the consistency of Einstein equations, and are the Israel junction conditions for this setup.

Note that in the case we will be mostly interested in of a static brane (with also ρ constant), the conservation equation is trivially satisfied and in this case the two jump equations are really independent and the equations simplify. One way of obtaining these equations would be to simply set $\dot{R} = \ddot{R} = 0$ into Eqs. (2.11)–(2.12). Alternatively, one can use the simple jump conditions derived by Binétruy *et al.* [32] for a general metric of the form

$$ds^2 = -n^2(t, r) dt^2 + a^2(t, r) d\Sigma_k^2 + b^2(t, r) dr^2, \quad (2.15)$$

where in our case $n = \sqrt{h(r)}$, $a = r/l$ and $b = 1/\sqrt{h(r)}$. The conditions for a static brane at $r = r_0$ are then simply [32]:

$$\frac{[a']}{a|b|} = -\frac{\kappa_5^2}{3}\rho, \quad \frac{[n']}{n|b|} = \frac{\kappa_5^2}{3}(2\rho + 3p), \quad (2.16)$$

where the above functions should be evaluated at the location of the brane and $[f']$ stands for the jump of the derivative of the function f around the brane: $[f'] = \lim_{\epsilon \rightarrow 0} (f'(r_0 + \epsilon) - f'(r_0 - \epsilon))$. For the above explained \mathbb{Z}_2 symmetric construction $[a'] = -2/l$, $[n'] = -h'/\sqrt{h}$, where we have assumed that for $r < r_0$ we are using the slice of the black hole metric that includes the singularity. Thus the jump conditions that a static brane has to satisfy are given by

$$6\sqrt{h(r_0)} = \kappa_5^2 \rho r_0 \quad \text{and} \quad 18h'(r_0) = -\kappa_5^4 (2 + 3\omega) \rho^2 r_0 \quad (2.17)$$

where we have defined $\omega = p/\rho$. Of course these equations agree with the more general jump equations (2.11)–(2.12). Note that the energy density on the brane, ρ , has to be positive to cut away the infinity.

2.3 The AdS–Schwarzschild black hole solutions

Let us now apply the jump equations for a static brane obtained above to the simple AdS–Schwarzschild black hole case (no vector field in the bulk). Our motivation is to find a vacuum solution with 4D Lorentz invariance on the brane and thus we will study static brane solutions ⁽⁴⁾ with a vanishing induced curvature on the three dimensional spatial sections ($k = 0$). We will postpone consideration of other metrics with maximally symmetric 4D sections to Section 2.6. In an AdS–Schwarzschild black hole space-time, the jump equations (2.17) are:

$$36 \left(\frac{r_0^2}{l^2} - \frac{\mu}{r_0^2} \right) = \kappa_5^4 \rho^2 r_0^2, \quad 36 \left(\frac{r_0^2}{l^2} + \frac{\mu}{r_0^2} \right) = -\kappa_5^4 (2 + 3\omega) \rho^2 r_0^2. \quad (2.18)$$

Combining these two equations we obtain an expression for the black hole mass μ :

$$\mu = -\frac{1}{24} \kappa_5^4 (1 + \omega) \rho^2 r_0^4. \quad (2.19)$$

The black hole singularity is shielded by a horizon if its mass, μ , is positive; otherwise there is a naked singularity in the metric at $r = 0$. This condition then translates into $\omega < -1$.

⁽⁴⁾ There is another way to obtain a flat brane solution since a Minkowski metric can be written as an open FRW space ($k = -1$) with a scale factor linear in the cosmic time. However, as it will become evident in our general analysis in Section 2.6, such a solution will not satisfy the jump equations except in some very special cases.

Note that this condition violates the positive energy condition on the brane. However, it is not immediately obvious, whether such an exotic matter would necessarily lead to an instability. $\omega < -1$ could for example be obtained if in addition to a positive brane tension there is a small negative matter energy density (which is not accompanied by pressure, that is $\omega = 0$ for this component). Just as a negative tension brane at an orientifold type fixed point is stable, this violation of the null energy condition at the \mathbb{Z}_2 symmetric brane might not be harmful. This issue clearly calls for further investigations. As a corollary of (2.19), one can also see that with an ordinary brane tension ($\omega = -1$), one can not obtain a horizon⁽⁵⁾. Moreover, even for the equation of state $\omega < -1$, the solution is fine-tuned. The reason is that from the jump equations (2.18) μ can be expressed in terms of the AdS length l as

$$\frac{\mu}{r_0^4} = \frac{1}{l^2} - \frac{\kappa_5^4 \rho^2}{36}, \quad (2.20)$$

which when substituted back into (2.19) gives the relation⁽⁶⁾

$$\rho = \sqrt{\frac{-72}{1+3\omega}} \frac{1}{l\kappa_5^2}, \quad (2.21)$$

which is a fine-tuning for the size of the energy density on the brane, and is the analog of the Randall-Sundrum finetuning condition for the brane tension of the positive tension brane. Thus in the presence of the \mathbb{Z}_2 symmetry the case with a simple AdS-Schwarzschild black hole requires fine-tuning, and is not adjusting its vacuum energy to zero.

2.4 Self-tuning AdS-Reissner-Nordström black holes

We have seen above that the simple AdS-Schwarzschild black hole solution requires fine tuning. We will now show that the situation is different for the AdS-Reissner-Nordström black hole. First we note that there are no additional jump conditions due to the presence of the bulk gauge field, as long as the SM matter living on the brane is not charged under this gauge symmetry. The reason is that the Maxwell equations in the bulk are first order differential equations in terms of the field strength F_{AB} . Thus there are no second derivatives appearing in the equation, and as long as there are no delta function sources for this gauge field on the brane (which is ensured with the above assumption of SM not transforming under the bulk U(1) gauge symmetry), there will be no additional jump conditions. One may wonder how this fits with the fact that the only non-vanishing component of the field-strength tensor is F_{tr} , which from the point of view of the underlying vector field A_M is obtained by taking a derivative with respect to r , and naively seems to be odd under the \mathbb{Z}_2 symmetry. The point is that, to have an action \mathbb{Z}_2 invariant, one has to choose the different components of A_M to have different properties under the \mathbb{Z}_2 . However, the \mathbb{Z}_2 parity of A_r is a matter of choice. The Lagrangian (2.1) will be invariant under the \mathbb{Z}_2 symmetry with both positive or negative parity assignments for A_r . If we want to avoid the appearance of an extra jump condition for the gauge fields one should choose A_r to be even and $A_{t,x}$ to be odd under the \mathbb{Z}_2 parity. Then

⁽⁵⁾A static brane in a Schwarzschild black hole background with a horizon was found in [33]. However, this brane is not 4D Lorentz invariant but of positive spatial curvature ($k = +1$).

⁽⁶⁾ Of course this relation makes sense only if $\omega < -1/3$. For other equations of state, it is impossible for the brane to remain static in an AdS-Schwarzschild black hole space-time.

F_{tr} is even, and we recover the above conclusion that there is no additional jump condition for the gauge field. In fact, such vector fields are indeed present in supergravity theories [34]. However, with such a parity assignment there is an additional Chern-Simons term allowed in the action. The AdS RN solution remains a solution in the presence of this extra term in the Lagrangian, but it will likely not be the most general solution anymore. It would be worthwhile to study the most general solutions in the presence of the additional Chern-Simons term that we are setting to zero in this paper.

The remaining part of this section is devoted to the embedding of a flat brane in a charged black hole background. Not only can such a solution be found without finetuning of any parameter of the action, but in some regions of the plane (ω, ρ) describing the brane, the singularity of the black hole will be protected by two horizons. We have summarized the results in Fig. 1.

The jump equations for a static brane embedded in a charged black hole space-time are now

$$36 \left(\frac{r_0^2}{l^2} - \frac{\mu}{r_0^2} + \frac{Q^2}{r_0^4} \right) = \kappa_5^4 \rho^2 r_0^2, \quad 36 \left(\frac{r_0^2}{l^2} + \frac{\mu}{r_0^2} - 2 \frac{Q^2}{r_0^4} \right) = -\kappa_5^4 (2 + 3\omega) \rho^2 r_0^2. \quad (2.22)$$

The existence of the charge as a second constant of integration allows us to evade the previous fine-tuning because the two jump equations simply fix the mass and the charge of the black hole (in the case of a flat brane, *i.e.*, $k = 0$) in terms of brane parameters and r_0 :

$$\mu = 3 \left(l^{-2} + \frac{1}{36} \kappa_5^4 \omega \rho^2 \right) r_0^4, \quad Q^2 = 2 \left(l^{-2} + \frac{1}{72} \kappa_5^4 (1 + 3\omega) \rho^2 \right) r_0^6. \quad (2.23)$$

When the parameter ω is too small, $\omega < -1/3$, the positivity of Q^2 requires

$$\rho \leq \rho_0 = \sqrt{\frac{-72}{1 + 3\omega}} \frac{1}{l \kappa_5^2}. \quad (2.24)$$

The black hole singularity at $r = 0$ will be hidden behind a horizon provided the inequality (2.7) is fulfilled, which, in terms of the equation of state and the energy density on the brane, reads⁽⁷⁾

$$l^{-2} \left(l^{-2} - \frac{1}{36} \kappa_5^4 \rho^2 + \frac{1}{24} \kappa_5^4 (1 + \omega) \rho^2 \right)^2 < \left(l^{-2} - \frac{1}{36} \kappa_5^4 \rho^2 + \frac{1}{36} \kappa_5^4 (1 + \omega) \rho^2 \right)^3. \quad (2.25)$$

We immediately see that with an ordinary brane tension equation of state, $\omega = -1$, this equation can never be satisfied. However, as long as $\omega \neq -1$, there may exist in general an interval for the energy density where the singularity is protected behind inner and outer horizons. The constraint (2.25) for the existence of horizons can be written as a quadratic inequality for the energy density,

$$4 + \frac{1}{36} l^2 \kappa_5^4 (1 + 6\omega - 3\omega^2) \rho^2 - \frac{1}{324} l^4 \kappa_5^8 \omega^3 \rho^4 < 0, \quad (2.26)$$

whose discriminant is $\Delta_2 = (1 + \omega)^3 (1 + 9\omega) \kappa_5^8 l^4 / 1296$. When the discriminant Δ_2 is positive, the quadratic equation will have two real roots. It can be checked that both roots are positive when $\omega \leq -1$, both are negative when $-1/9 \leq \omega \leq 0$ and only one is positive when $\omega \geq 0$. Since only positive roots can correspond to a physical value of ρ^2 , we conclude that if ω is

⁽⁷⁾Note that this inequality automatically implies that the mass of the black hole is positive.

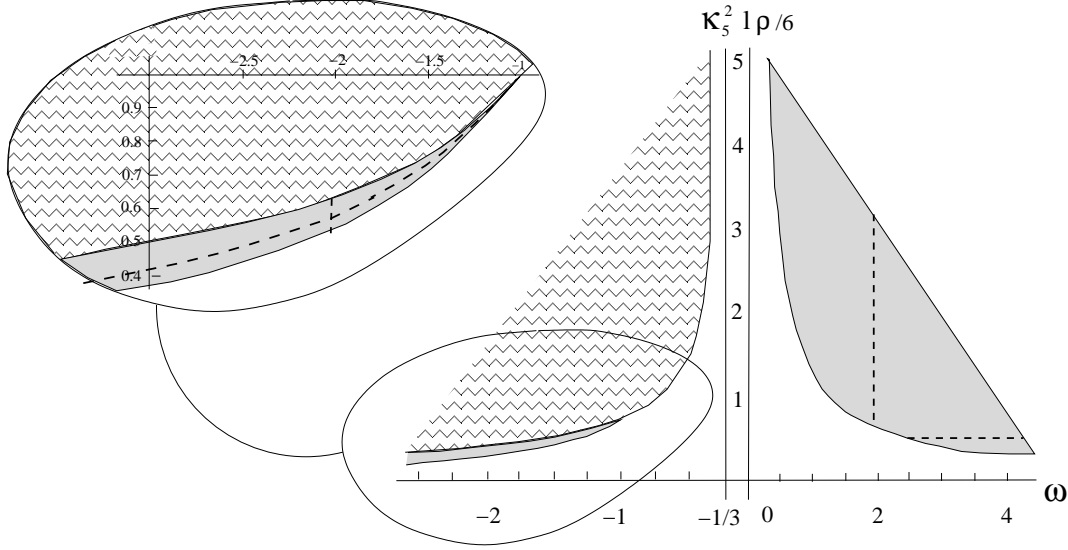


Figure 1: Phase diagram of a brane sitting in a bulk black hole background. In the upper left zig-zagged region, the brane will expand. In the other parts of the diagram, a brane can remain static and its induced metric is 4D Lorentz invariant. In the other filled regions, the black hole singularity is hidden by two horizons; however for $\omega > 0$ these horizons are developed in the part of the BH space-time cut by the \mathbb{Z}_2 orbifold. Fig. 2 shows the displacement of the horizons when moving along the dashed lines varying the energy density or the equation of state on the brane. The region $\omega < -1$ has been magnified.

positive, the inequality is satisfied for large enough value of the energy density while in the negative range, ω has to be less than -1 and the interval for the energy density where two horizons are developed is included in

$$\rho_- < \rho < \rho_+ \quad \text{with} \quad \rho_{\pm} = \frac{6}{l\kappa_5^2} \sqrt{\frac{1}{8\omega^3}(1 + 6\omega - 3\omega^2 \mp \sqrt{(1 + \omega)^3(1 + 9\omega)})}. \quad (2.27)$$

However this interval has to be reduced further since we need to require that Q^2 computed from (2.23) is positive, *i.e.*, $\rho < \rho_0 = \sqrt{-72/(1 + 3\omega)}/(l\kappa_5^2)$ which only partially overlaps with the allowed interval.⁽⁸⁾ In summary, the singularity at $r = 0$ will be protected by horizons *iff*

$$(\omega > 0 \quad \text{and} \quad \rho > \rho_-) \quad \text{or} \quad (\omega < -1 \quad \text{and} \quad \rho_- < \rho < \rho_0). \quad (2.28)$$

We still have to impose that these horizons sit between the singularity and the brane where the space is cut. This last condition translates in an upper bound for the parameter ω defining

⁽⁸⁾ In order to have two horizons, we also need the roots of the cubic equation associated to h to be positive. When the discriminant is negative, *i.e.*, the inequality (2.25) is satisfied, the cubic equation has three real roots x_1, x_2 and x_3 with the properties: $x_1 + x_2 + x_3 = 0$, $x_1x_2 + x_2x_3 + x_3x_1 = -l^2\mu/r_0^4$ and $x_1x_2x_3 = -l^2Q^2/r_0^6$. Since furthermore the sign of the cubic function at the origin is related to the sign of Q^2 , we conclude that in the interval (2.28), the black hole singularity will be shielded by *two* horizons. For $\omega < 0$ and $\rho > \rho_0$, only one horizon would be present but these solutions are non-physical since $Q^2 < 0$.

the equation of state. This condition is obtained by studying the position of the brane with respect to the positions of the horizons that correspond to the positive root of the cubic function $f(x) = x^3/l^2 - \mu x + Q^2$. It is worth noticing that the sign of f is changing when crossing a horizon but, from the first jump equation (2.17), we already know that f is positive at the brane. Moreover, $f'(0) = -\mu < 0$ and the sign of f' flips only between the two horizons. Thus it is enough to require that f' is positive at the brane, *i.e.*, $3r_0^4/l^2 - \mu > 0$ which from the expression of μ simply reads $\omega < 0$.

In conclusion, it is possible without any fine-tuning to embed a static brane in a charged black hole bulk. Moreover the singularity of this background will be hidden behind horizons if

$$\omega < -1 \quad \text{and} \quad \frac{6}{l\kappa_5^2} \sqrt{\frac{1}{8\omega^3}(1 + 6\omega - 3\omega^2 + \sqrt{(1 + \omega)^3(1 + 9\omega)})} < \rho < \sqrt{\frac{-72}{1 + 3\omega}} \frac{1}{l\kappa_5^2}. \quad (2.29)$$

Pictorially, the *phase diagram* of a brane in bulk black hole background is summarized in Fig. 1. Clearly, the same comments apply here for the $\omega < -1$ matter as for the AdS Schwarzschild case. We should note, that it is not clear whether or not the existence of such exotic matter is a generic requirement for a more complicated asymmetrically warped background (which could be obtained by introducing more fields into the bulk) to be self-adjusting. The above arguments seem to be specific enough to hope that a more complicated theory could avoid the presence of such matter.

So far we have only discussed solutions which include the black hole singularity and cutting away, by the \mathbb{Z}_2 orbifold symmetry, the region close to infinity. The motivation was to obtain an extra dimension of finite volume. The infinity of the black hole space-time asymptotes to AdS space, just like in a RS model with a single negative brane. This would give a divergent effective 4D Planck scale and therefore this possibility is excluded. However, as suggested in our previous analysis, the possibility to find a brane sitting between the singularity and the two horizons brings another way to prevent the 4D Planck scale to diverge: the singularity region is now cut away requiring a negative energy density on the brane but the space naturally ends at the inner horizon so that the 4D physics on the brane remains insensitive to the region beyond this inner horizon. Even though the sign in the first jump equation (2.17) flips, when squared the jump equations result in the same relation between the mass and the charge of the BH and the matter on the brane⁽⁹⁾

$$\mu = 3 \left(l^{-2} + \frac{1}{36} \kappa_5^4 \omega \rho^2 \right) r_0^4, \quad Q^2 = 2 \left(l^{-2} + \frac{1}{72} \kappa_5^4 (1 + 3\omega) \rho^2 \right) r_0^6. \quad (2.30)$$

Repeating our previous analysis we conclude now that the horizons belong to the part of the space that is kept *iff* $\omega \geq 0$. The constraint for these horizons to exist then becomes

$$\rho < -\frac{6}{l\kappa_5^2} \sqrt{\frac{1}{8\omega^3}(1 + 6\omega - 3\omega^2 + \sqrt{(1 + \omega)^3(1 + 9\omega)})}. \quad (2.31)$$

Note that, even if now $\omega > 0$, the second weak energy condition $\rho + p > 0$ is still violated. Thus the situation is actually quite similar to the previous construction and requires some exotic vacuum on the brane. Furthermore there is no way to consider these solutions as perturbations around the RS setup since the allowed region for the parameter ω is disconnected from the tension type equation of state. In the remaining part of the paper, we concentrate on the first type of solutions namely cutting the spacetime near infinity and keeping the region that includes the singularity.

⁽⁹⁾We thank Jim Cline and Hassan Firouzjahi for pointing out a sign error in the first version of this paper, which lead to an incorrect relation between the BH mass and charge, and the brane matter.

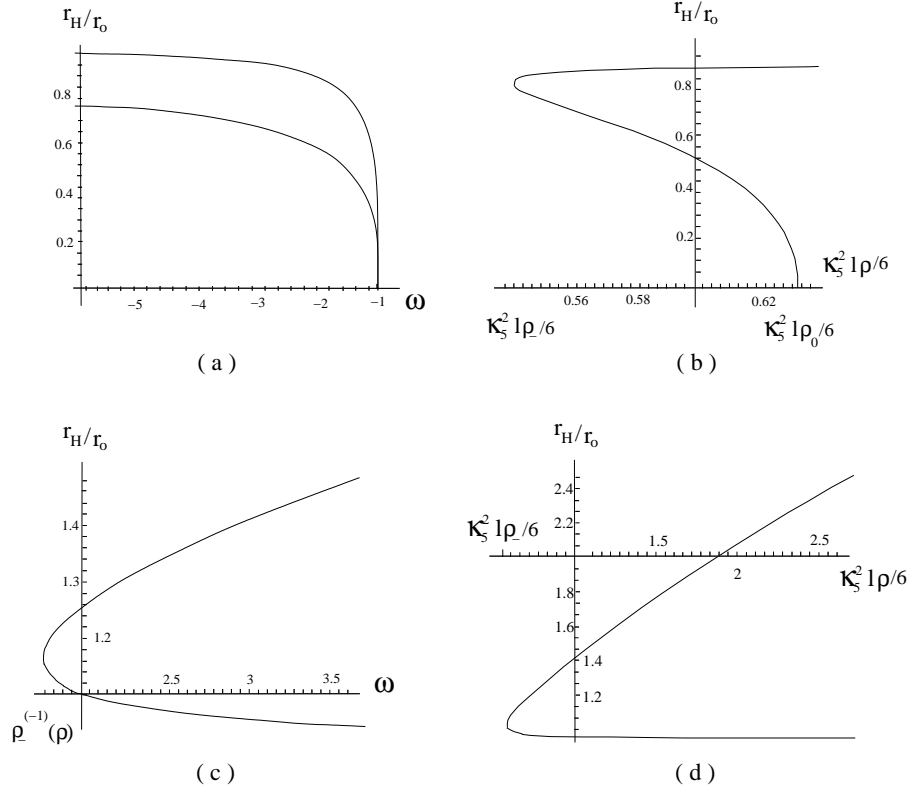


Figure 2: Displacement of the horizons when varying the energy density or the equation of state on the brane. Both the inner and outer horizons are shown in the plots. (a) ω is varying in the region $\omega \leq -1$. When ω reaches the critical point of a vacuum energy equation of state, the two horizons hit the black hole singularity; (b) the energy density is varying with $\omega < -1$ fixed. The two horizons are between the black hole singularity and the brane. When ρ goes to ρ_- , the two horizons degenerate and when ρ goes to ρ_0 , the inner horizon reaches the singularity; (c) ρ fixed, ω varying in the region $\omega > 0$; (d) $\omega > 0$ fixed, ρ varying. The two horizons belong to the cut region of the BH space-time. When ρ goes to ρ_0 , the two horizons degenerate.

2.5 Cutting the space at the horizon and the effective cosmological constant

We would like to show in this subsection that if we cut the space at the horizon, the 4D effective vacuum energy, computed as the integral of the action over the extra dimension, vanishes. This result may be expected because we were able to construct a solution to the Einstein equations which is 4D Poincaré invariant on the brane. However, unlike in other self-tuning solutions [7,8] where the space-time is cut at a naked singularity, the cut at a horizon is natural since from an observer's point of view it will take an infinite time to reach the horizon and boundary conditions at a horizon are well-defined and do not rely on the introduction of any *ad hoc* extra brane where some fine-tunings are reintroduced (see [10]).

So we would like to show that the following quantity is indeed vanishing:

$$\mathcal{I} = \int_{r_H}^{r_0} dr \sqrt{|g_5|} \left(\frac{1}{2\kappa^3} R - \frac{1}{4} F^2 - \Lambda_{bk} + \mathcal{L}_{\text{mat.}} \delta(\sqrt{g_{rr}}(r - r_0)) \right) \quad (2.32)$$

The first step is to evaluate the singular part coming from the brane namely to find the expression of $\mathcal{L}_{\text{mat.}}$ in terms of the 4D energy density and pressure that model the brane. As we have emphasized before, for the case where a horizon is present one needs an unconventional source with $\omega < -1$, which can not be obtained from stable dynamical fields, but should rather be thought of as a non-dynamical object. However, in order to calculate the relation between the energy densities and the original Lagrangian we assume that the results for the region $-1 < \omega < 1$ also apply for the case $\omega < -1$. In order to calculate the relation we are interested in we assume that the matter corresponds to a time-dependent scalar field

$$\mathcal{L}_{\text{mat.}} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi), \quad (2.33)$$

from which we obtain that

$$\mathcal{L}_{\text{mat.}} = p = -\frac{h'(r_0)}{\kappa_5^2 \sqrt{h(r_0)}} - \frac{4\sqrt{h(r_0)}}{\kappa_5^2 r_0}, \quad (2.34)$$

where we have used the jump equations (2.17) to derive the last equality. We emphasize again that for the case of matter with $\omega < -1$ (as we have in the case with a horizon) the assumption that the matter is dynamical like in (2.33) would imply an instability in the system, therefore one has to assume that rather than being a dynamical matter it is more like a topological object, similar to an orientifold. However, we assume that (2.34) holds even in this case.

In computing the bulk part of the integral, we must include the singular part of the curvature at the \mathbb{Z}_2 orbifold fixed point. To evaluate this singular contribution, we can use the form (2.8) of the bulk metric with $A(r) = \sqrt{h(r)}$, $B(r) = r$ and $C(r) = 1/\sqrt{h(r)}$; its curvature is given by [28] (for $k = 0$),

$$R = -\frac{2}{C^2} \left(\frac{A''}{A} + 3\frac{B''}{B} + 3\frac{B'^2}{B^2} + 3\frac{A'B'}{AB} - \frac{A'C'}{AC} - 3\frac{B'C'}{BC} \right) = -h''(r) - 6\frac{h'(r)}{r} - 6\frac{h(r)}{r^2}. \quad (2.35)$$

But because of the \mathbb{Z}_2 symmetry at $r = r_0$, the first derivatives are discontinuous which translates into delta function type singularities in the second derivatives. So the singular part of the curvature reads,⁽¹⁰⁾

$$R_{\text{sing.}} = \frac{4}{C^2} \left(\frac{A'}{A} + 3\frac{B'}{B} \right) \delta(r - r_0) = \left(2h'(r_0) + 12\frac{h(r_0)}{r_0} \right) \delta(r - r_0). \quad (2.36)$$

And finally, using the expression (2.4) of the bulk cosmological constant and the expression (2.6) of the gauge field strength, the effective cosmological constant becomes

$$\mathcal{I} = -\frac{1}{\kappa_5^2} \left[l^{-2} r^4 + \frac{Q^2}{r^2} \right]_{r_H}^{r_0} + \frac{1}{4\kappa_5^2} (2r^3 h' + 12r^2 h) \Big|_{r_0} + \frac{1}{2\kappa_5^2} (-r^3 h' - 4r^2 h) \Big|_{r_0} = \kappa_5^{-2} r_H^2 h(r_H) \quad (2.37)$$

which vanishes precisely because r_H is the position of the horizon. We can explicitly see that, as expected, this cancellation does not require adding anything at the horizon.

⁽¹⁰⁾ The sign is fixed by keeping the region of space-time between the singularity and the brane.

2.6 Maximally symmetric solutions on the brane

So far we have only considered flat brane solutions, but as we have discussed in the Introduction, it is important to look for other 4D maximally symmetric metrics on the brane, namely de Sitter or anti-de Sitter 4D space-time. In the 4D adjustment mechanism proposed by Hawking [35], a four form field provides a contribution to the cosmological constant whose magnitude is not determined by the field equations but appears as a constant of integration.⁽¹¹⁾ It was argued that its probability distribution may be exponentially peaked such that the flat solution will be preferred among the other maximally symmetric solutions. Unfortunately, Duff has shown [36] that the vanishing of the effective cosmological constant is the most unlikely possibility and the anthropic principle has to be invoked [37] to disentangle the different solutions. On the contrary, in the self-tuning approach to the cosmological constant on a brane, the 4D Minkowski metric provides the only maximally symmetric solution to Einstein's equations with an adequate choice of the conformal coupling to the brane [7]. We will argue now that self-selection of the flat brane solution remains true when the bulk scalar field is replaced by a vector field, without tuning parameters in the action.

De Sitter and anti-de Sitter solutions can be parametrized as FRW space-times,

$$ds^2 = -d\tau^2 + R^2(\tau)d\Sigma_k^2. \quad (2.38)$$

The explicit form of the scale factor can be deduced from the fact that (A)dS₄ space-time are solutions to the usual 4D Friedmann equation with a (negative) positive vacuum energy. This requires,

$$\frac{\dot{R}^2}{R^2} = \frac{1}{3}\kappa_4^2\Lambda_4 - \frac{k}{R^2} \quad (2.39)$$

Solving this ordinary differential equation leads to the following parameterizations [25, 38]:

$$\text{de Sitter: } \begin{cases} k = 0 & \text{and} & R(\tau) = R_0 e^{H\tau}; \\ k = -1 & \text{and} & R(\tau) = \sinh(H\tau)/H; \\ k = 1 & \text{and} & R(\tau) = \cosh(H\tau)/H. \end{cases} \quad (2.40)$$

$$\text{anti-de Sitter: } k = -1 \quad \text{and} \quad R(\tau) = \cos(H\tau)/H. \quad (2.41)$$

If such solutions exist, they must correspond to moving branes in the static bulk metric and thus must satisfy the jump equations (2.11)–(2.12). For a non-static brane, the second jump equation is equivalent to energy conservation, so $R(\tau)$ must satisfy,

$$\frac{\dot{R}^2}{R^2} = \frac{1}{36}\kappa_5^4\rho^2 - \left(\frac{k}{R^2} + \frac{1}{l^2} - \frac{\mu}{R^4} + \frac{Q^2}{R^6} \right) \quad \text{and} \quad \dot{\rho} + 3(1+\omega)\rho\frac{\dot{R}}{R} = 0. \quad (2.42)$$

When the parameter ω appearing in the equation of state is constant, the conservation equation becomes,

$$\rho = \rho_0 \left(\frac{R(\tau)}{R_0} \right)^{-3(1+\omega)}. \quad (2.43)$$

Plugging back into the first differential equation in (2.42), we determine in which cases the (A)dS space-time are solutions to the jump equations

⁽¹¹⁾For a modern version of this idea in a supersymmetric/superstring context, see [12].

- The 4D de Sitter metrics will be solution only when the equation of state on the brane corresponds to a vacuum energy, $\omega = -1$, and when this vacuum energy is bigger than the contribution from the bulk, in which case the Hubble parameter is given by $H^2 = \frac{1}{36}\kappa_5^4\rho^2 - l^{-2}$. All these solutions are moving in a pure Anti-de Sitter bulk, namely $\mu = 0$ and $Q^2 = 0$.
- The 4D Anti-de Sitter metrics will be solutions for a vacuum energy type brane ($\omega = -1$) with small enough energy density, in which case the Hubble parameter is given by $H^2 = l^{-2} - \frac{1}{36}\kappa_5^4\rho^2$ while the charge and the mass still vanish. These solutions are the AdS_4 branes in AdS_5 bulk studied in [39]. There are other 4D AdS solutions where the Hubble parameter is given by the 5D vacuum energy, $H^2 = l^{-2}$, and the energy density on the brane is balanced respectively by the (negative) mass of the 5D black hole if $\omega = -1/3$ ($\mu = -\frac{1}{36}\kappa_5^4 l^4 \rho_0^2$ and $Q = 0$) or by its charge if $\omega = 0$ ($Q^2 = \frac{1}{36}\kappa_5^4 l^6 \rho_0^2$ and $\mu = 0$).

This analysis shows that the 4D Minkowski metric is the only maximally symmetric solution on the brane up to, for particular equations of state, a discrete choice of the constants of integration. Hence, self-tuning in these models is somewhat natural: for a generic brane equation of state, the only continuous class of solutions is the set of flat brane solutions. This result offers a selection of a vanishing cosmological constant from symmetry requirements only. The vanishing of the cosmological constant is ensured by an adjustment of the charge and the mass of the black hole through gravitational waves emitted from the brane when a phase transition occurs for instance. A precise description of such a phase transition and the response of the bulk would however certainly deserve further scrutiny.

3 Lorentz violations in black hole backgrounds

We have seen in the previous section that asymmetrically warped spacetimes due to black holes in the bulk can exist, and perhaps even provide an adjustment mechanism for the effective cosmological constant. In this section, we investigate some of the physical consequences of such spacetimes. In this analysis we will not assume that the cosmological constant problem is resolved by these backgrounds, but we are rather interested in general consequences of having an asymmetrically warped metric. In particular, we will not need to have exotic energy densities on the branes (for which the price to pay is the reintroduction of fine-tuning in the theories). First we calculate the speed of gravitational waves based on the analysis of lightlike geodesics, then show how the graviton zero mode found by Randall and Sundrum would be modified in such spacetimes. Finally, we consider the cosmology of these models in order to gain some insight into the effective gravity observed on the brane.

3.1 Geodesics in the black hole background and the speed of gravitational waves

One of the most important consequences of an asymmetrically warped spacetime is that the speed of gravitational waves could differ from the speed of light. The reason for this is that in asymmetrically warped metrics the local speed of light is a function of the coordinate along

the extra dimension r :

$$c(r) = \left(\frac{h(r) l^2}{r^2} \right)^{1/2}. \quad (3.1)$$

Thus one might think of this spacetime as a medium with a continuously changing index of refraction $n(r)$, in which the propagation of light is the analog of the propagation of gravitational waves in the asymmetrically warped spacetimes. Propagation of light is governed by Fermat's principle, and if $c(r)$ is increasing away from the brane it may be advantageous for gravitational waves to bend into the bulk, and arrive earlier than waves propagating along the brane would. However since electromagnetism is forced to propagate along the brane, it will always keep the local velocity at the brane $c(r_0)$. Thus we expect as long as the velocity $c(r)$ increases away from the brane gravitational waves can travel faster than the local speed of light at the brane. This will lead to an apparent violation of Lorentz invariance due to the bulk dynamics in the low energy effective theory, and an apparent violation of causality from the brane observer's point of view, in the sense that gravitational waves emitted from a source will arrive earlier than light signals from the same source. We should emphasize that this is not a "real" violation of causality. Since there are no closed timelike curves in the theory, propagation of massless particles always proceeds forward in time. Apparent violation of causality simply means that the regions of spacetime which are in causal contact are different from what one would naively expect from an ordinary Lorentz invariant 4D theory. This is an experimentally verifiable prediction for these models, and can be tested by gravitational wave experiments like LIGO, VIRGO or LISA. Some of these points were made independently in Refs. [15–18, 25, 40] and it has been argued that this apparent Lorentz symmetry violation may provide an alternative to inflation to deal with the horizon problem. This is similar to another alternative to solve the horizon problem proposed in [41–43], where the speed of light is changing in time, thus regions of space-time may have been in causal contact even if they appear to be outside each others horizons. Violations of Lorentz invariance similar to the ones considered in this paper may also appear in non-commutative gauge theories [44, 45]. In generic non-commutative theories faster than light propagation will not be suppressed in the perturbative sector, and can not give a realistic model. In supersymmetric theories however the Lorentz violation in the perturbative sector vanishes, and will appear only as faster-than-light propagation in the solitonic sector, which is very weakly coupled to the perturbative states. This way interesting theories [46] analogous to the brane constructions presented here can be obtain from non-commutative gauge theories. However, it remains to be seen whether or not supersymmetry breaking will reintroduce the Lorentz breaking effects to the perturbative sector.

If $c(r)$ is a decreasing function as one moves away from the brane, for example in the case that there are horizons shielding the singularity from the brane, we expect that those gravitational waves that will be observed are those that remain on the brane, and thus no discrepancy between the speed of light and the speed of gravity should exist. It would be interesting to study the graviton wave equation in these cases, which we postpone until the next subsection. Below we show, that the analysis of lightlike geodesics is in complete agreement with the physical intuition sketched above.

Here we analyze the geodesics of the theories with asymmetrically warped metrics. Some of the features of geodesics in asymmetrically warped spacetimes have also been analyzed

in [15–17]. The equations for the geodesics are given by

$$\frac{d^2 x^M}{d\tau^2} + \Gamma_{PQ}^M \frac{dx^P}{d\tau} \frac{dx^Q}{d\tau} = 0, \quad (3.2)$$

where τ is an affine parameter along the trajectory (the proper time for the spacelike geodesic). As usual, these equations can be deduced from the Lagrangian provided by the proper distance. In the case of the black hole metric:

$$ds^2 = -h(r) dt^2 + l^{-2} r^2 d\vec{x}^2 + h(r)^{-1} dr^2, \quad (3.3)$$

the Lagrangian is explicitly given by:

$$\mathcal{L} = \frac{ds^2}{d\tau^2} = -h(r) \dot{t}^2 + l^{-2} r^2 \dot{\vec{x}}^2 + h^{-1}(r) \dot{r}^2, \quad (3.4)$$

where the dot represents differentiation with respect to the affine parameter τ . Furthermore the Euler–Lagrange equations are supplemented by the consistency condition defining the affine parameter:

$$\mathcal{L} = \epsilon, \quad \epsilon = -1/0/+1 \text{ for timelike/lightlike/spacelike geodesics, respectively.} \quad (3.5)$$

From the t - and \vec{x} -independence of the black hole metric, we find four Killing vectors and thus four corresponding conserved quantities:

$$\frac{\partial \mathcal{L}}{\partial \dot{t}} = -2h(r) \dot{t} = \text{const.} \equiv -2E, \quad (3.6)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}^i} = 2l^{-2} r^2 \dot{x}^i = \text{const.} \equiv 2p^i. \quad (3.7)$$

In terms of these conserved quantities, the consistency equation is

$$\dot{r}^2 + \frac{h(r)}{r^2} l^2 \vec{p}^2 - h(r) \epsilon = E^2. \quad (3.8)$$

As long as $\dot{r} \neq 0$, this equation is equivalent to the r component of the geodesic equations (3.2). However, for a straight geodesic parallel to the brane, *i.e.*, $\dot{r} = 0$, the first derivative of (3.8) also has to be satisfied.

From now, we will concentrate on lightlike geodesics only. The motion in the r direction transverse to the brane is then analogous to Newtonian dynamics for a particle of two units of mass with energy E^2 moving in the potential

$$V(r) = \frac{h(r)}{r^2} l^2 \vec{p}^2. \quad (3.9)$$

The behavior of $V(r)$ around the brane in the direction of the singularity, and the position of the brane with respect to the horizons if they exist, will determine the shape of the geodesics in the bulk.

From the jump equations (2.23), the expression for the potential becomes

$$V(r) = \vec{p}^2 \left(1 - \hat{\mu} r_0^4 / r^4 + \hat{Q}^2 r_0^6 / r^6 \right), \quad (3.10)$$

with

$$\hat{\mu} = l^2 \mu / r_0^4 = -3 \left(1 + \frac{1}{36} \kappa_5^4 \omega l^2 \rho^2 \right), \quad \hat{Q}^2 = l^2 Q^2 / r_0^6 = 2 \left(1 + \frac{1}{72} \kappa_5^4 (1 + 3\omega) l^2 \rho^2 \right). \quad (3.11)$$

The shape of the potential $V(r)$ will depend on the sign of $\hat{\mu}$ and whether there exists a minimum between the singularity and the brane. The condition for $\hat{\mu}$ to be positive is

$$(\omega > 0) \quad \text{or} \quad \left(\omega < 0 \text{ and } \rho < \rho_\mu = \sqrt{\frac{-36}{\omega l^2 \kappa_5^4}} \right). \quad (3.12)$$

And when $\hat{\mu} > 0$, the minimum of the potential will be at

$$r = r_0 \sqrt{\frac{3\hat{Q}^2}{2\hat{\mu}}}. \quad (3.13)$$

To locate the position of this minimum with respect to the brane, we can study the ratio $\hat{Q}^2/\hat{\mu}$, or equivalently, compute the *force*, $-V'(r)$, felt by the analogous Newtonian particle at the position of the brane:

$$-V'(r_0) = \frac{\vec{p}^2}{r_0} \left(-4\hat{\mu} + 6\hat{Q}^2 \right) = \frac{\vec{p}^2}{6r_0} (1 + \omega) l^2 \kappa_5^4 \rho^2, \quad (3.14)$$

from where we conclude that the minimum will be between the brane and the singularity *iff* $\omega < -1$. In summary, five different shapes for the potential arise and they are drawn in Fig. 3. This also shows that based on the intuitive picture presented at the beginning of this section one would expect the speed of gravitational waves can be larger than the speed of light when $\omega \geq -1$, since this is the case when the speed of light away from the brane increases.

Finally, we calculate the average speed of gravitational waves as observed from the brane observer's point of view. This can be calculated for the case when the speed of light away from the brane increases by first calculating the turning point of the Newtonian particle in the potential $V(r)$, which is the largest solution⁽¹⁾ between the singularity ($r = 0$) and the brane ($r = r_0$) of the following equation:

$$\left(\frac{E}{|\vec{p}|} \right)^2 = \frac{h(r_T)}{r_T^2} l^2. \quad (3.15)$$

Once the turning point is obtained (numerically), one can eliminate the proper time by dividing the expressions for \dot{x} by the expression for \dot{r} , and integrate the resulting equation to obtain $x(r)$. With this the distance on the brane x_{ret} after which the geodesic returns to the brane can be expressed as

$$x_{ret}(E/|\vec{p}|) = \int_{r_T}^{r_0} \frac{2l^2 dr}{r^2 \sqrt{(E/|\vec{p}|)^2 - h(r)l^2/r^2}}, \quad (3.16)$$

⁽¹⁾Note that we have cut the space at the brane and kept the region between the singularity and the brane. Since the potential diverges at the origin, there always exists a point where $\dot{r} = 0$ at which the potential is equal to the energy. This solution corresponds to the turning point where the geodesic starts moving back to the brane.

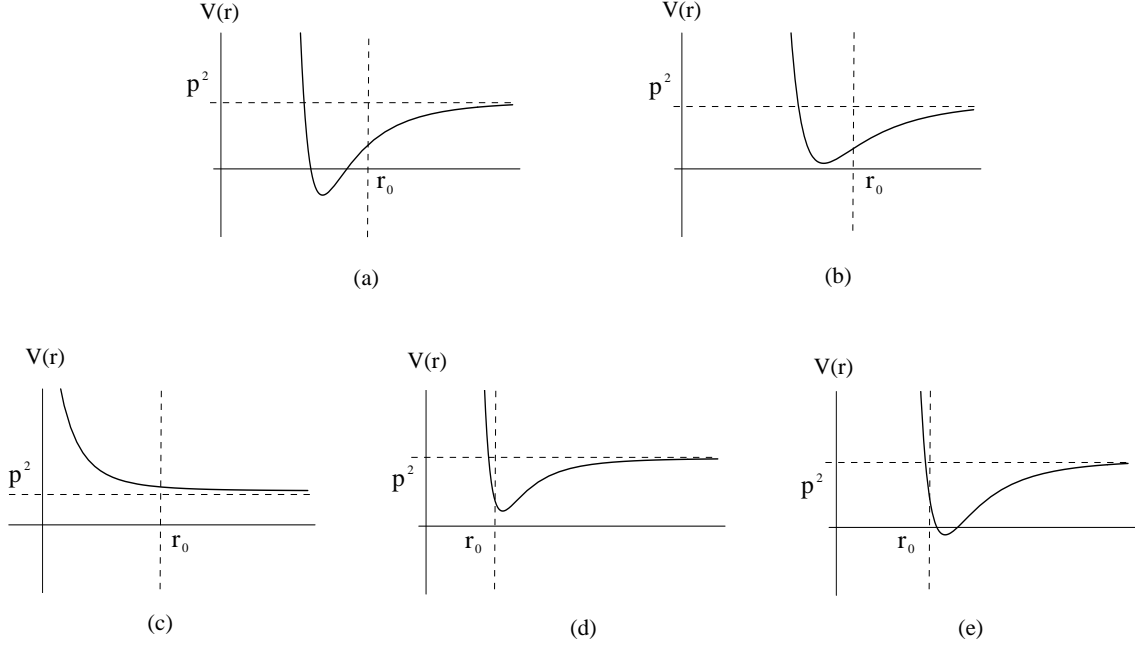


Figure 3: Shapes of the Newtonian potential. The functions ρ_- , ρ_0 and ρ_μ defined in (2.27), (2.24) and (3.12) border the different regions in the plane (ω, ρ) . The asymptotic value of the potential at infinity corresponds to the momentum $|\vec{p}|^2$ along the brane. The space is however cut at the brane sitting at $r = r_0$. We identify five different shapes for the potential. (a) $\omega < -1$ and $\rho_0 > \rho > \rho_-$: the potential vanishes at the two horizons. The geodesics will come back to the brane but, from a brane observer's point of view, it already takes an infinite time to cross the horizon and so the geodesics will never be seen as coming back. (b) $\omega < -1$ and $\rho_- > \rho$. The 4D local speed of graviton starts decreasing when the geodesic leaves the brane. (c) $-1 < \omega < 0$ and $\rho > \rho_\mu$. (d) $-1 < \omega < 0$ and $\rho_\mu > \rho$ or $0 < \omega$ and $\rho_- > \rho$. (e) $0 < \omega$ and $\rho > \rho_-$. In the three last configurations, the 4D speed of the graviton is increasing when going into the bulk, and on its return the geodesic may reach a point on the brane where light emitted with the graviton has not yet arrived.

and similarly the time it takes to return can be expressed as

$$t_{ret}(E/|\vec{p}|) = \int_{r_T}^{r_0} \frac{2 E/|\vec{p}| dr}{h(r) \sqrt{(E/|\vec{p}|)^2 - h(r) l^2 / r^2}}. \quad (3.17)$$

To obtain the relative speed compared to the speed of light, the average speed $c_{av} = x_{ret}/t_{ret}$ has to be compared with the local speed of light at the brane $c_{em} = (h(r_0) l^2 / r_0^2)^{1/2}$. One can clearly see from (3.17) that the average speed evaluated from the geodesics will depend on the value of $E/|\vec{p}|$, which is equivalent to choosing the oscillation length of the gravitational wave around the brane (which is also equivalent to how far into the bulk the gravitational wave is penetrating). The dependence of the average speed on the oscillation length is given in Fig. 4 for the cases where the speed of light away from the brane is increasing (that is cases (c), (d)

and (e)) of Fig. 3. The full gravitational propagation would be given by a superposition of these simple oscillating modes.

Depending on the exact structure of the low-energy effective theory, the Lorentz violation due to bulk effects may be transmitted to particles on the branes by gravitational loops. At the scale where gravity becomes strongly interacting, *i.e.* at the fundamental Planck scale, M_* , such loops will not be suppressed. In order for particle physics to remain Lorentz invariant, the Lorentz violations should decrease as the energy scale is increasing, otherwise one would expect unsuppressed Lorentz violating operators due to gravitational loops. One can see that in most cases considered here such constraints can be generically satisfied. The reason can be understood from a holographic argument, similar to ones explained in [47]. As the energy scale is increasing, gravity on the brane is probing less and less of the bulk region around the brane, indeed gravitational waves simply will have less time to travel further into the bulk. Thus in order to avoid large Lorentz violating operators being generated through gravitational loop effects one has to arrange that the region around the brane at a distance of the order M_*^{-1} should be very close to ordinary AdS space. Of course this can always be achieved by moving the black hole far away from the brane. In the language of the geodesic analysis of this section this constraint would imply that at small distances the speed of gravitational waves should approach the speed of light on the brane. One can see that the cases (c) and (e) in Fig. 4 automatically satisfy this requirement, and thus by adjusting r_0 one can satisfy experimental constraints on Lorentz violations in particle physics.⁽²⁾

3.2 A perturbative zero mode

We have seen from the analysis of the geodesics that one expects gravity to propagate with a speed different from ordinary electromagnetism for asymmetrically warped spacetimes. This can cause apparent violations of causality and Lorentz invariance in the gravitational sector, without affecting particle physics (except through gravitational loops).

Next we show an analysis different from the geodesic approach to demonstrate the same effect. We will examine how the graviton zero mode of the Randall-Sundrum model is modified in one of the black hole spacetimes considered in this paper. To simplify the equations, we will consider the black hole metric as a linearized perturbation around the RS spacetime. We have chosen to analyze the perhaps simplest form of matter on the brane, the case which simply corresponds to a brane tension ($\omega = -1$). As we have seen before, in this case there is no horizon, but there is a naked singularity away from the brane. In order to be able to analyze this theory as a perturbation around the RS metric, we have to cut the space-time with a second (fine-tuned) brane before the deviation from the RS spacetime becomes large (and of course before the appearance of the naked singularity). This way we obtain a metric that is close to the RS metric everywhere, and we will think of the mass and charge of the black hole as a perturbative expansion around the RS solution. In order to make this expansion more transparent, we first transform the black hole metric by an appropriate rescaling of the coordinates t and \vec{x} and a coordinate transformation $r = r_0 \exp(-ky)$, where $k = 1/l$, to the form

$$ds^2 = -e^{-2k|y|} \hat{h}(y) dt^2 + e^{-2k|y|} d\vec{x}^2 + \hat{h}(y)^{-1} dy^2, \quad (3.18)$$

⁽²⁾We thank John Terning for discussions on this issue.

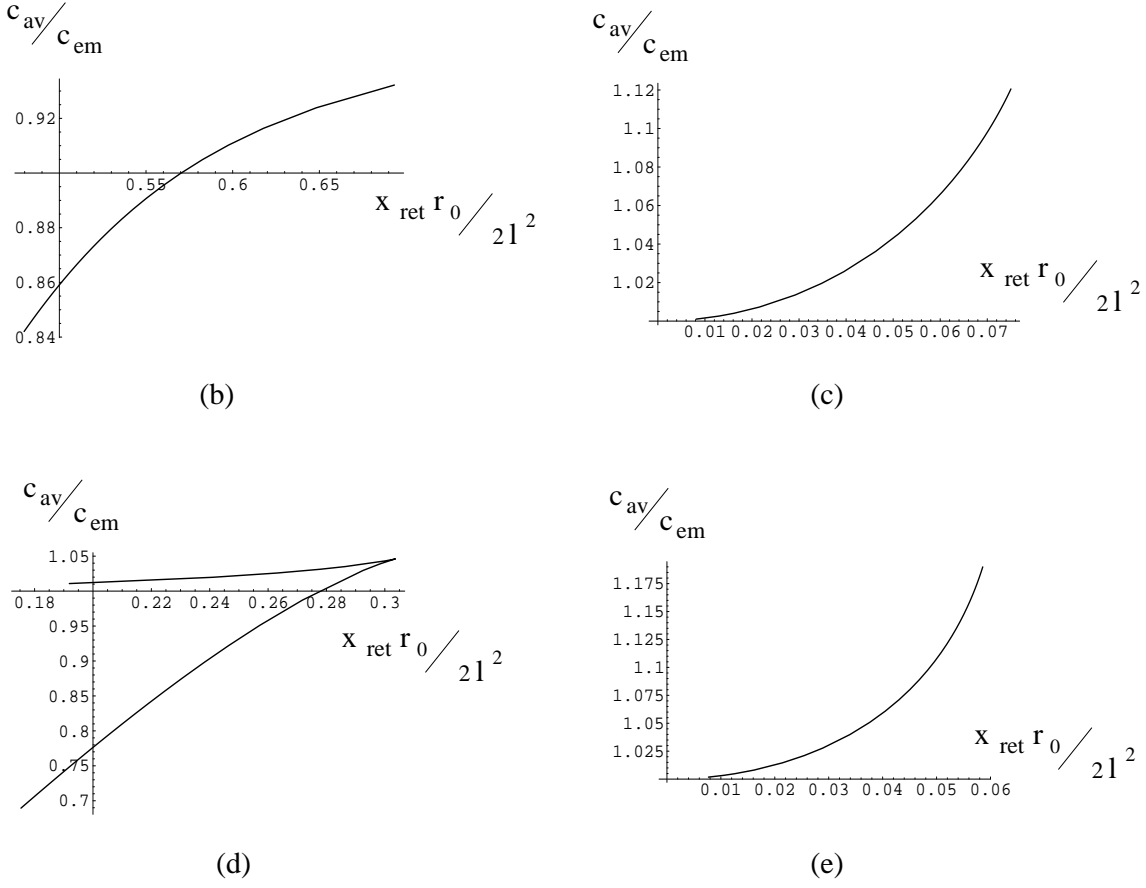


Figure 4: The Average speed of gravitons propagating along a geodesic off the brane as function of the distance on the brane. We clearly see that in the region of the plane (ω, ρ) where the Newtonian potential behaves like the shapes (c), (d) and (e) of Fig. 3, the graviton can propagate faster than the light and its speed increases with the distance to the source on the brane. We also note in the case (d) that there are various ways for gravity to propagate between the same points on the brane, *i.e.* different geodesics in the bulk with different values of $E/|\vec{p}|$ can return to the same point on the brane. When $E/|\vec{p}|$ is too large, then the average speed becomes lower than the speed of light. In the case (b), the graviton will always propagate along the brane with a speed faster than that in the bulk.

with $\hat{h}(y) = 1 - \mu l^2 r_0^{-4} e^{4ky} + Q^2 l^2 r_0^{-6} e^{6ky}$. The location of the brane is now at $y = 0$. As stated above, we also assume that $\mu l^2 r_0^{-4} e^{4ky}$ and $Q^2 l^2 r_0^{-6} e^{6ky}$ remain small everywhere in the bulk, and thus $\hat{h}(y)^{-1}$ can be expanded in μ and Q^2 . Now we would like to solve the linearized Einstein equation in this background, and in particular find the modified propagation speed of the graviton zero mode. One can show that the transverse traceless modes of the graviton

$h_{\mu\nu}$ satisfy the following linearized equation:

$$\begin{aligned} & -\frac{1}{2}D_M D^M h_{\mu\nu} + \frac{1}{2}R_{M\mu} h_\nu^M + \frac{1}{2}R_{M\nu} h_\mu^M - R_{\mu\rho\nu\sigma} h^{\rho\sigma} + \frac{1}{2}R_{\rho\sigma} h^{\rho\sigma} g_{\mu\nu} - \frac{1}{2}R h_{\mu\nu} \\ & = \frac{1}{2}\kappa_5^2 F_{\rho M} F_\sigma^M h^{\rho\sigma} g_{\mu\nu} - \frac{1}{4}\kappa_5^2 F_{MN} F^{MN} h_{\mu\nu} - \kappa_5^2 \Lambda_{bk} h_{\mu\nu}, \end{aligned} \quad (3.19)$$

where the covariant derivatives are with respect to the background metric g_{MN} , as are the Riemann tensor and the Ricci tensor and scalar. We have also checked that these transverse traceless modes decouple from perturbations of the gauge field and thus only the background gauge field appears in the right hand side of the equation. In fact, one can show that after explicitly including the expressions for the covariant derivatives and the background quantities in (3.19) the equation simply becomes identical to the equation for a minimally coupled massless scalar in the bulk, just like in the case of the RS-type backgrounds [48]. Hence, we simply study the propagation of a scalar field Φ in the bulk. We know that for vanishing μ and Q^2 the zero mode solution in the RS model is just $\Phi = \Phi_0 e^{i(\omega t - \vec{q} \cdot \vec{x})}$, where $\omega^2 = q^2$, and Φ_0 is a normalization constant. Thus we look for a perturbative solution of the bulk equation of the form,

$$\Phi = \Phi_0 (1 + \delta\Phi(y)) e^{i((\omega + \delta\omega)t - \vec{q} \cdot \vec{x})}, \quad (3.20)$$

where $\delta\Phi(y)$ and $\delta\omega$ are assumed to be proportional to μ and Q^2 . A scalar zero mode of a similar form has been found for another asymmetrically warped metric in Ref. [18]. Expanding the linearized equation around the background (3.18) one obtains,

$$\delta\Phi'' - 4k\delta\Phi' + \mu l^2 r_0^{-4} q^2 e^{6ky} - Q^2 l^2 r_0^{-6} q^2 e^{8ky} + 2\delta\omega q e^{2ky} = 0. \quad (3.21)$$

As explained above, we are assuming for the sake of perturbativity that the space-time is made finite by the introduction of a second brane, and does not include the singularity. Therefore we need to impose a boundary condition on $\delta\Phi$ that its derivative at the two branes vanishes, *i.e.*

$$\delta\Phi'|_{y=0, y_R} = 0, \quad (3.22)$$

where y_R is the position of the regulator brane. Enforcing these boundary conditions one obtains the dispersion for the zero mode, which is given by

$$\delta\omega = \frac{1}{4}q \left(-2\mu l^2 r_0^{-4} + Q^2 l^2 r_0^{-6} (1 + e^{2ky_R}) \right) e^{2ky_R}. \quad (3.23)$$

This dispersion relation remains linear such that the speed of propagation of gravitational waves is constant and given by

$$c_{grav} = 1 + \left(-\frac{\mu l^2}{2r_0^4} + \frac{Q^2 l^2}{4r_0^6} (1 + e^{2ky_R}) \right) e^{2ky_R}, \quad (3.24)$$

which has to be compared with the propagation speed of electromagnetic waves given by $c_{em} = \sqrt{\hat{h}(y=0)} = 1 - \frac{1}{2}\mu l^2 r_0^{-4} + \frac{1}{2}Q^2 l^2 r_0^{-6}$ in these coordinates. For the $\omega = -1$ spacetimes μ and Q are related by the formula $\mu = \frac{3}{2}Q^2/r_0^2$, therefore the difference of the two speeds can be written as

$$c_{grav} - c_{em} = (\cosh(2ky_R) - 1) e^{2ky_R} \frac{Q^2 l^2}{2r_0^6} \geq 0, \quad (3.25)$$

Therefore the gravitational speed for the propagation of the zero mode is always larger than the speed of electromagnetic waves in the scenario considered here. The LIGO experiment may be able to detect gravitational waves from type II supernovae up to a distance [49] of about 20 Mpc ($\sim 6 \cdot 10^7$ ly). For objects of such a distance even a tiny difference in the speeds of gravitational and electromagnetic waves would cause a huge time difference, and thus the possible values of μ and Q could be severely constrained [18]. In fact, the limitations of such measurements are likely not to lie in the time resolution of the gravitational and the light signal, but rather the opposite problem: if there is an appreciable difference in the propagation speeds then due to the huge distance to the expected sources the arrival time differences could turn out to be way too big to be able to identify the fact that the source for the gravitational wave and the light was the same. For a supernova 20 Mpc away from us, and very conservatively assuming that the arrival time difference should be less than 5 years, in order to be able to actually detect the different arrival times one needs to have the difference in the speeds to be less than $\frac{\delta c}{c} \leq 10^{-7}$. Otherwise the gravitational wave experiments will simply not be able to identify the source for the observed gravitational waves. Type I supernovae could likely be detected by LIGO only if they happen within our galaxy. These are very rare, however assuming the best possible scenario one could see a supernova a few hundred thousand light years from us. In this case (again assuming a very conservative time difference of 5 years) the maximum value of $\frac{\delta c}{c}$ that could be tested is of the order of 10^{-3} .

3.3 Cosmological Expansion and Effective Gravity Theory

So far we have discussed the interesting physical consequences of asymmetrically warped space-times: the possible difference in the speeds of gravitational and electromagnetic waves, and the possible adjustment of the cosmological constant to give a flat brane. Next we would like to understand what kind of gravitational theory a 4D observer on the brane would see. There are at least two distinct possibilities:⁽³⁾ the effective action for gravity could explicitly break the 4D Lorentz invariance for instance by introducing different coefficients for time and space derivatives in the kinetic terms of the graviton, or the effective action may only violate the weak equivalence principle by the presence of some extra fields which couple differently to matter and gravitation forcing the graviton to propagate differently as the other gauge bosons (see [50] for a review on the different tests of Einstein gravity). Note that the distance of the brane to the singularity or the horizon could be such a field. Both approaches will manifest themselves experimentally by observing different speeds of propagation for the graviton and the photon. Instead of trying to understand this (very important) question of how to incorporate the Lorentz violating effects into the low energy effective action, we solve the simpler problem (which still gives us some insights into the long distance behavior of gravity) of finding the cosmological evolution of the brane in the presence of matter perturbations on top of the vacuum energy. We will express the sources on the brane as the sum of the (non-dynamical) vacuum energy, plus the matter perturbations:

$$\rho_{tot} = \rho_0 + \rho, \quad p_{tot} = p_0 + p. \quad (3.26)$$

⁽³⁾We thank Nemanja Kaloper for discussions on this issue.

The expansion of the brane can then be read off from (2.13)-(2.14). Using (2.22) the form of the expansion equation can be simplified to

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{\kappa_5^4 \rho_0}{18} \rho + \mu \left(\frac{1}{R^4} - \frac{1}{r_0^4}\right) - Q^2 \left(\frac{1}{R^6} - \frac{1}{r_0^6}\right) + \frac{\kappa_5^4}{36} \rho^2, \quad (3.27)$$

while the conservation of energy equation is given by the slightly unconventional expression

$$\dot{\rho} + 3(\rho_0(1 + \omega_0) + \rho(1 + \omega)) \frac{\dot{R}}{R} = 0. \quad (3.28)$$

In these equations, R is nothing but the scale factor on the brane and a dot denotes a derivative with respect to the proper time on the brane, *i.e.*, the usual FRW time; r_0 is the position of the brane before matter was introduced. On comparison with similar terms in usual 4D Friedmann equation, from (3.27) we can identify the 4D effective Planck scale as

$$\frac{1}{M_{Pl}^2} = \frac{\kappa_5^4 \rho_0}{6}. \quad (3.29)$$

If one assumes that energy densities on the brane are of the order of the TeV scale, then the required size of the five dimensional Planck scale would be given by $M_* = 10^8$ GeV, with $\kappa_5^2 = 1/M_*^3$. One can also see that very close to the static solution (that is at $R = r_0$ there is no correction to the ordinary FRW equation, which one could perhaps interpret as the zero mode of this model reproducing ordinary 4D gravity. However, as the brane moves further away from the static point, the corrections to the Friedmann equation will start becoming sizeable. This has one positive consequence: a cosmological constant term $\mu/r_0^4 - Q^2/r_0^6$, the usual dark radiation term μ/R^4 and due to the charge of the black hole a contribution that is similar to the contribution of a 4D massless scalar, Q^2/R^6 is obtained, which can be used to fit the Friedmann equation to the observed accelerating Universe [24]. This possibility has also been pointed out by Refs. [22, 23]. However, these terms also pose a problem: if μ, Q^2 and r_0 are such that the cosmological constant gets adjusted to zero for $R = r_0$, then after just a short expansion period the above mentioned new terms in the Friedmann equation will start dominating, if μ and Q^2 remain constant. In order to overcome this problem, one has to assume that the adjustment mechanism that sets the cosmological constant dynamically to zero also operates during the ordinary expansion of the Universe, thus making μ and Q^2 time dependent. Let us determine how small the characteristic time scale for the adjustment mechanism would have to be in order for the solution to track the vanishing cosmological expansion solution. In such a case the corrections due to the change in μ of the form $\dot{\mu}/R^4$ should cancel the corrections from the change in the position of the brane of the form $\mu \dot{R}/R^5$, (up to remaining terms in the Friedmann equation of order $H^2 = (\dot{R}/R)^2$). This would lead to $\dot{\mu}/\mu \sim \dot{R}/R \sim H$; thus, the characteristic time scale for the adjustment should be of the order or shorter than the Hubble scale. If one wants an ordinary FRW Universe after nucleosynthesis then one should require that this scale is shorter than H_{BBN} . This is not a very strong restriction for the time scale of adjustment, and could still leave the possibility for early inflation open. Of course if one is not trying to solve the cosmological constant problem, then μ, Q^2 and r_0 should be viewed as free parameters that are not necessarily related to the fundamental Planck scale. In this case these parameters can be simply used to fit the observed accelerating Universe without

assuming any time dependence for them. In that case the acceleration of the expansion of the Universe would be a manifestation of gravitational Lorentz violations in extra dimensions rather than a consequence of a tiny bare vacuum energy in four dimensions.

4 Conclusions

The work of Randall and Sundrum has revealed that new physics can emerge from a non-trivial (“warped”) geometry mixing the four dimensions of our brane-world with an extra non-compact dimension. While the main attention has been focused on solutions preserving 4D Lorentz symmetry in the bulk, other solutions exist with different warp factors for timelike and spatial directions. In fact these *asymmetric* solutions are the more generic ones, and they open up new perspectives to low energy gravitational interactions due to the Lorentz symmetry violation they can mediate.

One of the most striking features is the possibility for gravitational waves to propagate faster than electromagnetic waves stuck on the brane [15–18]. This apparent violation of causality from a brane observer point of view could be experimentally tested by gravitational wave detectors. In the same vein, we have shown that the addition of a vector field in the bulk alleviates the fine-tuning of the cosmological constant problem. The bulk has a geometry of an AdS–Reissner–Nordström black hole, where the relaxation could be achieved by the adjustment of the mass and charge of the black hole. A horizon could protect the black hole singularity, however the existence of a horizon requires the existence of some exotic energy density on the brane. The cosmology in asymmetrically warped spacetimes can explain the observed acceleration of the Universe which thus would appear as a manifestation of gravitational Lorentz violations in extra dimensions. If one wants to solve the cosmological constant problem and explain the accelerating Universe simultaneously, then one has to assume that the relaxation of the mass and charge is of the order or faster than the Hubble scale.

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